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1979 D R HALVERSON, G L WISE F49620-77-C-0101

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20. ABSTRACT

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ASYMPTOTICALLY OPTIMUM ZERO MEMORY DETECTORS FOR DEPENDENT NOISE PROCESS

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Abstract

Design of detectors for known signals in non-Gaussian ϕ-mixing noise is considered. Applying the criterion of asymptotic relative efficiency, the design of the optimal memoryless detector is specified and is seen to depend only on second-order statistical knowledge of the noise. It is then shown that in many cases this design reduces to approximating the noise process with an m-dependent process and obtaining the optimal nonlinearity through a limiting process. In addition, conditions are given for the existence of a unique optimal nonlinearity.

I. Introduction

The notion of optimality of a detector has been well formulated in terms of the Neyman-Pearson criterion [1]. There is currently increasing interest in the case where the noise is non-Gaussian and dependent; in such a situation Neyman-Pearson techniques are frequently intractable. Because of this, it has been found necessary to adopt an alternative fidelity criterion, commonly that of asymptotic relative efficiency (ARE). We consider memoryless discrete time detection of signals in corrupting noise. Results have been recently obtained [2] for the case where the noise is m-dependent; the resultant detectors guarantee performance at least as good as that under a "white noise" assumption. The assumption of an m-dependent noise model is, however, in several ways overly restrictive, and we have therefore chosen to extend the results of [2] to the much broader class of symmetrically ϕ-mixing noises. This class consists of processes which exhibit a characteristic rolloff in dependency as samples are more widely separated in time, and includes the m-dependent processes.

II. Preliminaries

Let $\{N_i;\ i=1,2,\ldots\}$ be a strictly stationary sequence of random variables. For $a\le b$, define $M(a,b)=\sigma\{N_a,N_{a+1},\ldots,N_b\}$, the σ-algebra generated by the indicated random variables, where a and b may take on extended real values. Then $\{N_i;\ i=1,2,\ldots\}$ is <u>symmetrically ϕ-mixing</u> if there exists a nonnegative sequence $\{\phi_i;\ i=1,2,\ldots\}$ with $\phi_i\to 0$ such that for each k, $1\le k<\infty$, and for each $i\ge 1$, $E_1\in M(1,k)$, $E_2\in M(k+i,\infty)$ together imply $|P(E_1\cap E_2)-P(E_1)P(E_2)|\le\phi_i \min\{P(E_1),P(E_2)\}$. We will consider detection of a known constant positive signal s in an additive symmetrically ϕ-mixing noise process, where we observe realizations $\{y_i;\ i=1,\ldots,n\}$ of the process $\{Y_i;\ i=1,\ldots,n\}$ and where $\{N_i;\ i=1,2,\ldots\}$ is a symmetrically ϕ-mixing noise process. For convenience, we assume the existence of joint densities $f_j(\cdot,\cdot)$ of N_k and N_{k+j}, with

$$K_n^*(x,y)\triangleq\sum_{j=1}^{n} h_j(x,y)/\sqrt{f(x)f(y)} \text{ square integrable}$$

for all n, where $h_j(x,y)\triangleq f_j(x,y)+f_j(y,x)$, and we will assume the common univariate density $f(\cdot)$ is differentiable and strictly positive on the real line with finite Fisher's information number $I(f)\triangleq\int(f'(x))^2/f(x)\,dx$ (all integrals, unless indicated otherwise, are taken over the entire real line). As in [2] we will optimize over the class of optimal memoryless detectors under a "white noise" assumption, i.e. where a test statistic $T_g(y)=\sum_{i=1}^{n} g(y_i)$ is compared to a threshold. Specifying the choice of g will therefore be of prime concern.

We will restrict the class $\mathscr{G}$ of nonlinearities g to include those measurable real-valued functions for which we can find $s_1>0$ such that the random variable $g(N_1+s)$ is second-order for all $s\in[0,s_1]$, and such that the following mild regularity conditions hold, where $s_n=K/\sqrt{n}$ for some positive constant K and $E_s(\cdot)$ denotes expectation computed under H_1 with signal strength s (by proper choice of the threshold T, we assume without loss of generality that the random variables $g(N_i)$ are zero mean):

(a) $\frac{\partial}{\partial s}E_s\{T_g(Y)\}|_{s=0}>0$ for all n

(b) $$\lim_{n\to\infty}\frac{\left[\frac{\partial}{\partial s}E_s\{T_g(Y)\}|_{s=0}\right]^2}{n\,E_0\{[T_g(Y)]^2\}}\triangleq\eta(g)>0$$

(c) $$\lim_{n\to\infty}\int g(x)f'(x-s_n)dx=\int g(x)f'(x)dx$$

(d) $$\lim_{n\to\infty}\frac{E_{s_n}\{[T_g(Y)-E_{s_n}\{T_g(Y)\}]^2\}}{E_0\{[T_g(Y)]^2\}}=1$$

(e) $$\frac{\partial}{\partial s}\int g(x)f(x-s)dx|_{s=0}=\int\frac{\partial}{\partial s}g(x)f(x-s)|_{s=0}\,dx$$

(f) $$\sigma_0^2(g)\triangleq E\{g(N_1)^2\}+2\sum_{j=1}^{\infty}E\{g(N_1)g(N_{j+1})\}>0.$$

We remark that the restrictions on the densities $f_j(\cdot,\cdot)$, $f(\cdot)$, and the class $\mathscr{G}$ are no more

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restrictive than those imposed for the m-dependent case [2]. Properties (a)-(d) are assumptions conventionally imposed for application of the Pitman-Noether theorem, whereas (e) is an exceedingly mild restriction. For a large class of noise processes, including all of the examples of [2], property (f) is automatically satisfied and may therefore be ignored.

From here on we will frequently refer to "Condition A" defined by

Condition A: $\{N_i;\ i=1,2,\ldots\}$ is symmetrically ϕ-mixing with $\sum_{i=1}^{\infty} \phi_i^{\frac{1}{2}} < \infty$.

The following shows that $\sigma_0^2(g)$ is well-defined and will allow us to benefit from the asymptotic nature of the ARE criterion:

Proposition 1: Assume that Condition A is satisfied and that $E\{g(N_1)\}=0$, $E\{[g(N_1)]^2\} < \infty$ where g is measurable. Then $\sigma_0^2(g)$ converges absolutely, and $\sum_{i=1}^{n} \frac{g(N_i)}{\sqrt{n}} \xrightarrow{\mathcal{D}} N(0,\sigma_0^2(g))$.

Proof: The first part of the proposition follows as a direct corollary to Theorem 21.1 of [3], as does the latter part if $\sigma_0^2(g)>0$. If $\sigma_0^2(g)=0$ the latter part follows as a consequence of Lemma 3 of [3]. QED

We are now in position to consider the problem of determining necessary and sufficient conditions for the nonlinearity $g \in \mathscr{G}$ to be optimal. We first need the following:

Lemma 1: Assume $\{N_i;\ i=1,2,\ldots\}$ is symmetrically ϕ-mixing and suppose X is $M(1,k)$ ($M(k+i,\infty)$) measurable and Y is respectively $M(k+i,\infty)$ ($M(1,k)$) measurable. If h is a measurable real-valued function and Y and h(X) second-order, then $E\{|E\{Y|X\}-E\{Y\}|\cdot h(X)\} \leq 2\phi_i^{\frac{1}{2}}(E\{Y^2\})^{\frac{1}{2}}(E\{h(X)^2\})^{\frac{1}{2}}$.

Proof: Consider the case where X is $M(1,k)$ measurable and Y is $M(k+i,\infty)$ measurable. In the manner of [3] we may set $Y = \sum_j u_j I_{A_j}$ where $A_j \in M(k+i,\infty)$. Then

$$|E\{Y|X=x\}-E\{Y\}| = \Big|\sum_j u_j \int_{Y(A_j)} f_{Y|X}(y|x)dy - \sum_j u_j \int_{Y(A_j)} f_Y(y)dy\Big|.$$

Multiplying by $h(x)f_X(x)$, integrating, and applying the triangle and Schwarz inequalities give $E\{|E\{Y|X\}-E\{Y\}|\cdot h(X)\} \leq [E\{E\{Y^2|X\}+E\{Y^2\}\}]^{\frac{1}{2}}$

$$\cdot \Big[\int \sum_j |P(A_j|\{\omega : X(\omega)=x\})-P(A_j)|\cdot h(x)^2 f_X(x)dx\Big]^{\frac{1}{2}},$$

whence comparing 20.27 of [3] gives

$$E\{|E\{Y|X\}-E\{Y\}|\cdot h(Y)\} \leq 2\phi_i^{\frac{1}{2}}[E\{Y^2\}]^{\frac{1}{2}}[E\{(h(X))^2\}]^{\frac{1}{2}}.$$

The other case follows in a similar way using the fact that $\{N_i;\ i=1,2,\ldots\}$ is symmetrically ϕ-mixing. QED

We can now prove an important theorem.

Theorem 1: Assume that Condition A is satisfied and that $g \in \mathscr{G}$. Then g is optimal (in the sense of the ARE) if and only if g is a solution (up to a scale factor) of

$$-\sum_{j=1}^{\infty} \int h_j(x,y)\, g(y)dy + \tilde{g}(x) = f(x)g(x) \qquad (1)$$

where $\tilde{g}(x) = -f'(x)$.

Proof: We consider the efficacy $\eta(g) = [\frac{\partial}{\partial s} E_s\{g(Y_1)\}|_{s=0}]^2/\sigma_0^2(g)$ and its role in the ARE as given by the Pitman Noether theorem [4]. Proceeding in the manner of [2] we will obtain necessary and sufficient conditions to maximize $H(g) = -\int g(x)f'(x)dx - \lambda\sigma_0^2(g)$. Noting that we are maximizing H over zero-mean second-order $g(N_i)$, we may consider variations $\delta g(N_i)$ such that $(g+\varepsilon\cdot\delta g)(N_i)$ is zero-mean and second-order, hence $\delta g(N_i)$ is also zero-mean and second-order. Defining $J_g(\varepsilon) \triangleq H(g+\varepsilon\cdot\delta g)$, we then can show

$$J_g'(0) = -\int \delta g(x)f'(x)dx - 2\lambda\int g(x)\delta g(x)f(x)dx - 2\lambda \sum_{j=1}^{\infty}\int\int [g(y)f_j(x,y)+g(y)f_j(y,x)]\delta g(x)dydx.$$

Lemma 1 followed by the Tonelli and Fubini theorems implies

$$\sum_{j=1}^{\infty}\int\int [g(y)\delta g(x)f_j(x,y)+g(y)\delta g(x)f_j(y,x)]\, dydx = \int\Big(\sum_{j=1}^{\infty}\int [g(y)f_j(x,y)+g(y)f_j(y,x)]dy\Big)\delta g(x)dx.$$

Since a necessary condition for the optimality of g is $J_g'(0)=0$ for all admissable variations δg and for some λ, we note that varying λ scales g, and letting $\lambda=\frac{1}{2}$, we obtain (1). In the manner of [2], this condition is seen also to be sufficient. QED

The order of summation and integration in (1) is important and is a noteworthy example of a case when an integral and sum do not commute. Methods for obtaining the solution g are addressed in the section which follows.

III. The Optimal Nonlinearity

As we have remarked, we need consider solutions to (1). In the sequel, in view of Theorem 1, we will, up to a scale factor, identify "optimal" nonlinearities with solutions g of (1), even though one must also check that $g \in \mathscr{G}$. To approach the problem of obtaining g we need the following:

Lemma 2: Assume that Condition A is satisfied and $\{g_n\}$ is a sequence of elements of $\mathscr{G}$ such that $\{E\{[g_n(N_1)]^2\}\}$ is bounded. For $n>m$ let

$$R_{mn}(x) = \sum_{j=m+1}^{n} \int (h_j(x,y)/\sqrt{f(x)})\, g_n(y)\,dy, \text{ then}$$

$$\sup_{n>m} E\left\{(R_{mn}(N_1)/\sqrt{f(N_1)})^2\right\} \longrightarrow 0 .$$

Proof: $$|R_{mn}(x)/\sqrt{f(x)}| \le \sum_{j=m+1}^{n} |E\{g_n(N_{j+1})|N_1=x\}| + \sum_{j=m+1}^{n} |E\{g_n(N_1)|N_{j+1}=x\}| . \quad (2)$$

Lemma 1 implies

$$E\{|E\{g_n(N_{i+1})|N_1\}|\cdot|E\{g_n(N_{j+1})|N_1\}|\}$$
$$\le 2\phi_i^{\frac{1}{2}}(E\{(g_n(N_1))^2\})^{\frac{1}{2}}\cdot(E\{(E\{g_n(N_{j+1})|N_1\})^2\})^{\frac{1}{2}},$$

whence repeated application of Lemma 1 on $(E\{(E\{g_n(N_{j+1})|N_1\})^2\})$ K-1 times gives

$$(E\{(E\{g_n(N_{j+1})|N_1\})^2\})^{\frac{1}{2}} \le 2^{\sum_{k=1}^{K-1} 2^{-k}} \phi_j^{\sum_{k=2}^{K} 2^{-k}}$$
$$\cdot(E\{(g_n(N_1))^2\})^{\sum_{k=2}^{K} 2^{-k}} \cdot (E\{(E\{g_n(N_{j+1})|N_1\})^2\})^{2^{-K}} .$$

Passing to the limit as $K \to \infty$ gives

$$E\{|E\{g_n(N_{i+1})|N_1\}|\cdot|E\{g_n(N_{j+1})|N_1\}|\}$$
$$\le 4\phi_i^{\frac{1}{2}}\,\phi_j^{\frac{1}{2}}\, E\{g_n(N_1)^2\}. \quad (3)$$

Similarly we obtain

$$E\{|E\{g_n(N_1)|N_{i+1}\}|\cdot|E\{g_n(N_1)|N_{j+1}\}|\}$$
$$\le 4\phi_i^{\frac{1}{2}}\,\phi_j^{\frac{1}{2}}\, E\{g_n(N_1)^2\}. \quad (4)$$

In conjunction with the triangle inequality, (2)-(4) then imply

$$E\{(R_{mn}(N_1)/\sqrt{f(N_1)})^2\} \le 4 \sum_{i=m+1}^{n} \phi_i^{\frac{1}{2}}(E\{(g_n(N_1))^2\})^{\frac{1}{2}}.$$

We note that $\{g_n(N_1)\}$ has bounded second moment, from which the desired result follows. QED

Consider solutions g_m to

$$-\sum_{j=1}^{m} \int h_j(x,y)\,g_m(y)\,dy + \tilde{g}(x) = f(x)\,g_m(x). \quad (5)$$

Such solutions are not in general optimal unless the noise is m-dependent [2], however the g_m will lead us to the optimal nonlinearity g through convergence arguments. Recall [2] that (5) possesses an associated family of orthonormal eigenfunctions $\{\psi_{im}\}_i$ and eigenvalues $\{\lambda_{im}\}_i$. We can then show the following second-moment property of the $\{g_m\}$:

Proposition 2: Assume that Condition A is satisfied. If -1 is not a limit point of the double sequence $\{\lambda_{im}\}_{i,m}$, then $E\{(g_m(N_1))^2\}$ is bounded.

Proof: Under the hypothesis, we may assume m is large enough so that g_m uniquely exists [2]. Consider the solution g_m^λ to

$$-\sum_{j=1}^{m} \int h_j(x,y)\,g_m^\lambda(y)\,dy + \tilde{g}(x) = \lambda f(x)\,g_m^\lambda(x) \quad (6)$$

where $\lambda > 0$. There exists arbitrarily large λ such that a solution to (6) always exists uniquely. Multiplying by $g_m^\lambda(x)$, integrating, applying the Schwarz inequality and Lemma 1 of [3] we obtain

$$4 \sum_{j=1}^{m} \phi_j^{\frac{1}{2}}\, E\{(g_m^\lambda(N_1))^2\} + [E\{(\tilde{g}(N_1)/f(N_1))^2\}]^{\frac{1}{2}}$$
$$\cdot[E\{(g_m^\lambda(N_1))^2\}]^{\frac{1}{2}} \ge \lambda E\, (g_m^\lambda(N_1))^2\},$$

and hence $E\{(g_m^\lambda(N_1))^2\}$ is bounded over m for λ sufficiently large (recall $I(f) < \infty$). The solution g_m^λ of (6) satisfies [5]:

$$\sqrt{f(x)}\; g_m^\lambda(x) = \frac{\tilde{g}(x)}{\lambda\sqrt{f(x)}} - \frac{1}{\lambda}\sum_{i=1}^{\infty} \frac{c_{im}\lambda_{im}}{\lambda+\lambda_{im}}\, \psi_{im}(x)$$

where $c_{im} = \int [\tilde{g}(x)\psi_{im}(x)/\sqrt{f(x)}]\,dx$,

and therefore $\displaystyle\sum_{i=1}^{\infty} \frac{c_{im}^2\,\lambda_{im}^2}{(\lambda+\lambda_{im})^2}$ is bounded over m .

Defining $S_m = \{i: |\lambda_{im}| \le 2\lambda\}$ and noting that all terms are nonnegative, we see that $\sum_{i\in S_m} c_{im}^2\,\lambda_{im}^2$ and $\sum_{i\in S_m^c} c_{im}^2$ are bounded over m. But noting that $|1+\lambda_{im}| \ge \epsilon$, we see that

$$\sum_{i\in S_m} \frac{c_{im}^2\,\lambda_{im}^2}{(1+\lambda_{im})^2} \le \frac{1}{\epsilon^2} \sum_{i\in S_m} c_{im}^2\lambda_{im}^2 \text{ and}$$

$$\sum_{i\in S_m^c} \frac{c_{im}^2\lambda_{im}^2}{(1+\lambda_{im})^2} \le \left(\frac{2\lambda}{2\lambda-1}\right)^2 \sum_{i\in S_m^c} c_{im}^2 . \quad \text{Since [2],}$$

$$g_m(x) = \frac{\tilde{g}(x)}{f(x)} - \frac{1}{\sqrt{f(x)}} \sum_{i=1}^{\infty} \frac{c_{im}\lambda_{im}}{1+\lambda_{im}} \psi_{im}(x),$$ we

obtain the desired result from the triangle inequality. QED

An important relationship between the $\{g_m\}$ and the optimal nonlinearity g may be seen in the following (where from here on, unless stated otherwise, the g_m are regarded as operating on N_1):

Proposition 3: Assume that Condition A is satisfied. If a subsequence $\{g_{m_k}\}_{k=1}^{\infty}$ of $\{g_m\}_{m=1}^{\infty}$ satisfies $g_{m_k} \xrightarrow{m.s.} g$, then g is optimal.

Proof: We have that g_{m_k} satisfies

$$0 = -\int \delta g(x) f'(x)dx - 2\lambda \int g_{m_k}(x)\delta g(x) f(x)dx$$

$$- 2\lambda \sum_{j=1}^{m_k} \iint g_{m_k}(y)\ h_j(x,y)\delta g(x)dxdy \qquad (7)$$

where $\delta g(N_1)$ is a zero mean, second-order variation. Application of the Schwarz inequality gives

$$\lim_{k\to\infty} \int g_{m_k}(x)\delta g(x)dx = \int g(x)\delta g(x)f(x)dx.$$

Application of the triangle inequality with Lemma 1 of [3] shows

$$\lim_{k\to\infty} \sum_{j=1}^{m_k} \iint g_{m_k}(y)\ h_j(x,y)\delta g(x)dxdy =$$

$$\sum_{j=1}^{\infty} \iint g(y)\ h_j(x,y)\delta g(x)dxdy.$$

The result follows now from the proof of Theorem 1 by taking limits of both sides of (7). QED

The following provides conditions for the existence of a unique optimal nonlinearity and specifies how it is related to the $\{g_m\}$:

Proposition 4: Assume that Condition A is satisfied. If -1 is not a limit point of $\{\lambda_{im}\}_{i,m}$ then there exists a unique (up to a scale factor) optimal nonlinearity g, and $g_m \xrightarrow{m.s.} g$.

Proof: Note that the comments prefacing the proof of Proposition 2 apply here. For g_n and g_m with n>m, setting $z_{nm}=g_n-g_m$ gives

$$-\int K_m^*(x,y)z_{nm}(y)\sqrt{f(y)}dy-R_{mn}(x)=z_{nm}(x)\sqrt{f(x)}.$$

Consider solutions z_{nm}^{λ} to

$$-\int K_m^*(x,y)z_{nm}^{\lambda}(y)\sqrt{f(y)}dy-R_{mn}(x)=\lambda z_{nm}^{\lambda}(x)\sqrt{f(x)}. \quad (8)$$

Thus z_{nm}^{λ} is a solution to

$$-\sum_{j=1}^{m}\int h_j(x,y)z_{nm}^{\lambda}(y)dy - \sum_{j=m+1}^{n}\int h_j(x,y)g_n(y)dy$$

$= \lambda f(x)z_{nm}^{\lambda}(x)$. If we multiply each side of the above by $z_{nm}^{\lambda}(x)$ and integrate, Lemma 1 of [3] in conjunction with Proposition 2 imply, for large fixed λ, $\sup_{n>m} E\{(z_{nm}^{\lambda}(N_1))^2\} \to 0$.

But (8) implies

$$\sqrt{f(x)}\ z_{nm}^{\lambda}(x) = -\frac{R_{mn}(x)}{\lambda} - \frac{1}{\lambda}\sum_{i=1}^{\infty}\frac{d_{imn}\lambda_{im}}{\lambda+\lambda_{im}}\psi_{im}(x), \quad (9)$$

where $d_{imn} = \int(-R_{mn}(x))\psi_{im}(x)dx$.

Lemma 2 implies $\sup_{n>m} \sum_{i=1}^{\infty} d_{imn}^2\lambda_{im}^2/(\lambda+\lambda_{im})^2 \to 0$

as $m \to \infty$. We then use the dominating techniques of the proof of Proposition 2 to obtain

$\sup_{n>m} \sum_{i=1}^{\infty} d_{imn}^2\lambda_{im}^2/(1+\lambda_{im})^2 \to 0$ as $m \to \infty$. But applying (9) with $\lambda = 1$ in conjunction with Lemma 2 gives $\sup_{n>m} E\{(z_{nm}(N_1))^2\} \to 0$. In view of Proposition 3, the Cauchy criterion then establishes existence and mean-square convergence. An analogous argument for the equation $-\sum_{j=1}^{\infty}\int h_j(x,y)h(y)dy = f(x)h(x)$ shows $h(N_1) = 0$ almost surely, from which uniqueness follows. QED

IV. Interpretation of Results

Before condensing the preceding results, it is beneficial to note the corresponding result for the m-dependent case. The following theorem is a summary of the relevant work of Poor and Thomas [2].

Theorem 2: Suppose the noise is m-dependent. We can make the following statements:

(a) If all the eigenvalues of $\{\lambda_{im}\}_{i=1}^{\infty}$ are not equal to -1, then a unique optimal nonlinearity g_m exists and is given by

$$g_m(x) = \frac{\tilde{g}(x)}{f(x)} + \sum_{i=1}^{\infty}\frac{c_{im}\lambda_{im}}{1+\lambda_{im}} \cdot \frac{\psi_{im}(x)}{\sqrt{f(x)}}$$

where $c_{im}=\int[\tilde{g}(x)\psi_{im}(x)/f(x)]dx$ and $\tilde{g}(x)=-f'(x)$.

(b) An optimal nonlinearity g_m exists if and only if for any $\lambda_{im} = -1$ we have $c_{im} = 0$.

Existence and uniqueness of the optimal nonlinearity thus depends on the information contained in the eigenvalues $\{\lambda_{im}\}_{i=1}^{\infty}$. As we have seen, the same thing is true in the more general symmetrically ϕ-mixing case, and we could state a theorem analogous to Theorem 2 for the ϕ-mixing case by imposing requirements on the $\{\lambda_{im}\}_{i,m}$ to insure that we would have the appropriate statement corresponding to (b). The difficulty with this is that such a condition would necessarily involve relative convergence of the c_{im} and $1+\lambda_{im}$ to zero, and would therefore be extremely difficult to check. Since the integral equation given by Theorem 1 is in most cases difficult to solve, it would be most useful to specialize our results to the case where the solution can be found as a limit of functions which are obtained from standard Hilbert-Schmidt techniques. We will say that a nonlinearity g is <u>reachable</u> if there exists a subsequence $\{g_{m_k}\}_{k=1}^{\infty}$ of $\{g_m\}_{m=1}^{\infty}$ which converges in mean-square to g. Because the g_m satisfy (5), the closed form expressions of [2] are applicable; moreover, a reachable nonlinearity g is approximately equal to g_{m_k} for large k.

We then may summarize our results for the symmetrically ϕ-mixing case as follows:

<u>Theorem 3</u>: Assume that Condition A is satisfied. We can make the following statements:

(c) If -1 is not a limit point of $\{\lambda_{im}\}_{i,m}$ then there exists a unique optimal nonlinearity g given by $g_m \xrightarrow{m.s.} g$ (and thus g is reachable).

(d) There exists a reachable optimal nonlinearity g if and only if g is the mean-square limit of a subsequence of $\{g_m\}_{m=1}^{\infty}$.

<u>Proof</u>: This follows directly from Propositions 3 and 4. QED

V. Conclusion

We have shown how the optimal detector may be designed for memoryless detection of signals in a large class of additive ϕ-mixing noise. We have seen that such a design effectively reduces to detector design under an asymptotic m-dependent assumption, whereby standard Hilbert-Schmidt techniques may be employed. This design has the further advantage of requiring only second-order statistical knowledge of the noise.

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